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A Time Series analysis of U.K. Construction and Real Estate Indices

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Abstract This study assess the nonlinear behavior of U.K. construction and real estate indices. Standard unit root tests show that both time series are $I(1)$ processes. However, the empirical results show that the returns series for both indices deviate from the null hypothesis of white noise. Moreover, we have found evidence of nonlinearity but strong evidence against chaos for the returns series. Further tests show that the source of nonlinearity is rather different. Hence, the construction index returns series displays weak nonlinear forecastability, typical of nonlinear deterministic processes, whereas the real estate index could be characterized as a stationary process about a nonlinear deterministic trend.

Keywords Nonlinearity · Heteroskedasticity · Random walk · Chaos · Nonlinear predictability

1 Introduction

The random walk hypothesis (RWH) of asset prices posists that prices traded in a market that is efficient cannot be predicted by using historical price information. This implies, therefore, that prices traded in such a market are serially uncorrelated. The behaviour of security prices, in particular, in the context of a weak form efficient market, has been, and continues to, engage the attention

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of academics and practitioners and regulators. While academicians seek to understand the behaviour of security prices over time, practitioners and investors are mainly interested in any observed exploitable patterns while regulators on the other hand are interested in the informational efficiency of the securities market. The knowledge of the behaviour of asset prices is of considerable interest to a large number of interest groups. Resources continue to be spent in studying the behaviour of asset prices with the view to enhancing current understanding.

Most of the past studies of the behaviour of United Kingdom stock market prices have accepted weak form market efficiency (see Kendall (1953), Brealey (1970), Dryden (1970), Cunningham (1973) among others). Recent advances in mathematical modelling have sparked a large volume of research into the re-examination of the behaviour of security returns. A large number of recent studies have applied much more sophisticated techniques to examine the behaviour of financial series in recent times (see Lo and MacKinlay, 1988, 1989; Liu and He, 1991; Scheinkman and LeBaron, 1989; Hsieh ,1991; Willey, 1992; Poon and Taylor, 1992; Abhyankar et al., 1995, 1997; Opong et al, 1999; Wright, 2000; Belaire-Franch, 2003; Belaire-Franch and Opong, 2002 among others).

Most of the studies that have used advanced modelling techniques to examine the properties of financial variables are United States based. Few of such studies have used U.K. sector data. A major contribution of this current study, therefore, is to add to the small but growing amount of evidence concerning the behaviour of returns in the U.K. Equity market. This study is important for a number of reasons. First, the U.K. Market is among the world's major stock markets and, therefore, understanding the behaviour of returns of assets traded in the market is a worthwhile venture. Second, evidence about what happens in the U.K. Market will permit comparison with studies done elsewhere. Third, if asset returns can be modelled in the U.K. Stock market, it will challenge or even invalidate weak form market efficiency. As argued by Belaire-Franch and Opong (2002) if asset returns could be modelled, it may also imply that stock returns could be predicted if the specific form of the underlying price structure can be determined; such information will be of obvious benefit to investors.[1] Fourth, this study uses sector indices i.e. Real Estate and Construction indices which may behave differently from the FTSE All Share Index or FTSE 100 Index which are normally the focus of most of the research on time series properties of equity returns. In its statement of principles in section 2.1(a), Guide to calculation methods for the UK series of FTSE actuaries share indices, the FTSE argues that: '...the indices and index statistics are produced primarily for use in analysing investment strategies and as a measure of portfolio performance for professional investors such as pension funds, insurance companies and other institutional investors'. There is the presumption that index series provide economic value and there

[1] It must be pointed out that any potential benefit derived from such knowledge may be short lived since it is likely to be competed away.

is anecdotal evidence that practitioners, particularly investment analysts, do use market indices in their investment decision making. FTSE's statement of principles suggest that market practitioners from among both investor and brokers community are actively involved in determining the 'best practice' to be used in the calculation of the indices so as to meet the needs of the market. Given that the stated primary purpose of an index is to reflect the movement in the underlying market for its constituents, knowledge of the time series properties of the construction and real estate index series should provide a profitable opportunity to property and real estate investors if the movement in the series are exploitable. The index series used in this study are in themselves not publicly traded. However, and as pointed out earlier, FTSE which is the creator of the index series heavily involve investment practitioners in their calculation to meet the needs of the market. Therefore, even though the series are not publicly traded, private contracts based on the index series can readily be obtained in the market. Indeed a number broking houses offer a contract for difference based on any sector index. Sector indices are therefore important investment tools in the arsenal of portfolio managers, pension funds and institutional investors for managing their investment risks. In the context of weak form market efficiency, knowledge of movements in the index series should not provide any worthwhile investment opportunity in the presence of transaction costs. The dearth of evidence regarding the behaviour of sector indices and their growing importance in private derivative trading provide a major motivation for this study.

Lastly the behaviour of a sectoral index (especially Real Estate and Construction) are of major interest given that most households largest lifetime investment are likely to be in real estate. Given the points raised above, this study contributes to the extant literature, and may have policy implications regarding the future introduction of index options based on the index series in this study.[2]

This paper is organized as follows. Next section reviews the literature concerning empirical studies on weak efficiency in the stock market. Section 3 is devoted to a description of the data. Sections 4 and 5 explore the hypothesis of unit root/stationarity of the indices (in logs) and the hypothesis of random walk, respectively. Section 6 analyses the issue of nonlinearity in mean and chaos for the indices returns, whereas nonlinear forecastability of the returns is assessed in section 7. Section 8 concludes.

2 Previous studies

Recent studies that have applied modelling techniques to examine the behaviour of asset returns in the U.K. are very few. Commercial Real Estate company returns are examined by Kleiman, Payne and Sahu (2002) for random walk behaviour. Using standard unit root tests, Kleiman et al. report

[2] Currently there is no options trading on the Real Estate and Construction Indices.

that the RWH cannot be rejected for the European, North American and Asian markets they examined. They conclude that opportunities exist for short-lived diversification benefits but these disappear in the long run and conclude that international commercial Real Estate markets are weak form efficient. Garino and Sarno (2004) construct a housing demand function and test the hypothesis that for a 20-year period there have been speculative bubbles in the U.K. house prices using data from Halifax House Price Survey database. They report that U.K. house prices exhibit non-stationary characteristics.

Payne and Sahu (2004) examine the behaviour of the World Real Estate Index and the World Stock Market Index. They use the Phillips and Peron (1988) unit root tests, the Cochrane (1988) variance ratio test and Johansen and Juselius (1990) co-integration tests. Their reported results indicate support for random walk price behaviour in the securites market. Connock (2002) tested for evidence of serial correlation in U.K. house prices using data from the Nationwide and Halifax house price surveys. The reported results show evidence of serial correlation and argue that it could provide potential arbitrage opportunities. However, given transaction costs, such observed serial correlations may not be exploitable.

Brooks and Tsolacos (2001) tested for both short run and long run variations in Real Estate returns. They report that, in the short run, there are potential excess returns but caution that if transaction costs are taken into account, excess return will be zero. The persistence in UK Real Estate returns and its implications for market efficiency has also been tested by Lee and Ward (2000). They report that there is persistence in Real Estate returns which in theory cast doubt on market efficiency. They conclude, however, that no economic gains can be achieved if transactions costs are taken into account. Brooks and Tsolacos (2001) tested for random walk behaviour in property company returns in addition to their test of the relationship between property returns and interest. They report that the first differences of property log returns can be characterised as a random walk .

Macgregor and Schwann (2003) examined the short run co-movements between differing property types and regions. They argue that a core of a small number of distances lead to variations in real estate returns. These disturbances are transmitted between regions and property types. They applied Phillips-Peron unit root tests to examine whether Real Estate returns were stationary. They report that each property type has at least one common cycle and that common cycles were transmitted across regions very rapidly. They conclude that there is commonality of returns across regions and this has implications for investors as it limits portfolio diversification.

3 The data

Data for the study is obtained from Datastream and cover the period from from 23rd March 1999 to 31st December 2009. The index series are constructed from the underlying traded securities on the London Stock Exchange. ThomsonReuters is the creator of the FTSE Indices which are publicly available.[3] The indices are constructed based on the Industry Classification Benchmark (ICB) to define sectors. Real estate and construction firms come under classification 8600 and 2300 respectively.[4] The FTSE Policy Group whose membership is representative of users of the FTSE Indices has been established by FTSE as an independent committee responsible for overseeing and maintaining the ground rules for the management of the FTSE UK index series. The Policy Group ensures that a consistent approach is applied to the selection of constituents. The industry sector or super sector may change from time to time. The reassessment of the industry sector or super sector of a constituent firm is made by the FTSE Global Classification Committee (for industry sector) or by FTSE (for super sectors). The meeting to review constituents is held on the Wednesday after the first Friday in March, June, September and December. FTSE's ground rule 8.2 provides that periodic changes to the industry classification of a company to be agreed and announced by The FTSE Global Classification Committee and implemented after the close of the index calculation on the third Friday of the month in which a meeting is held.[5]

In this study, we use the FTSE indices FT32RL£(for Real Estates) and FTA3S3£(for Construction) firms respectively which are based on ICB super sector codes 8600 and 2300 respectively and which are constructed from the underlying traded securities. FTSE provide a detailed methodology for the index calculation. Briefly stated, the indices are arithmetic weighted indices where the weights are the market capitalisation of each company. The daily index value is the total market value of all companies within the index divided by a devisor. The divisor is usually set at 100 at the start of the index calculation and adjusted over time for capitalisation changes to the constituents thus allowing the index value to remain comparable over time. Thus the value of an index is given by:

$$I = \Sigma_{i=1}^{n} \frac{p_i \times s_i \times f_i}{d}.$$

where I is the index value; n = number of securities in the index; p, the closing price for the constituent security; s, outstanding shares in issue for the security; f, free float factor expressed as a number between 0 and 1 where 1 represents free float (free float factor for each security which is published by FTSE) and d is the divisor that represents the total issued share capital of the index. The divisor is adjusted to allow for capital changes for the constituent firms.

[3] http://www.ftse.com/Indices/UK_Indices/Downloads/AppendixB_Reference_Codes.pdf (Accessed 10th November 2010.)

[4] ibid.

[5] FTSE Calculation methodology can be found at: http://www.ftse.com/Indices/UK_Indices/Downloads/uk_calculation.pdf (Accessed 10th November 2010).

4 Unit root tests

In this section, we test two kinds of closely related null hypothesis. On the one hand, we test whether each index series, in logs, is nonstationary by means of the unit root tests recently developed by Ng and Perron (2001). Those are the M tests developed in Perron and Ng (1996) to allow for GLS detrending of the data. The Modified Information Criteria (MIC) along with GLS detrended data yield a set of tests with desirable size and power properties.

On the other hand, we test the null hypothesis of stationarity by means of the so-called KPSS test by Kwiatkowski et al. (1992). In order to estimate the long run variance, we will use alternative kernels, and alternative automatic data-dependent procedures to choose the value for the bandwith parameter.

4.1 Unit root and stationarity tests

Given a time series y_t, $t = 0, \ldots, T$, assume that the observations are generated by

$$y_t = d_t + u_t$$

where d_t is a deterministic term and

$$u_t = \alpha u_{t-1} + v_t$$

Under the null hypothesis of unit root, $\alpha = 1$.

Perron and Ng (1996) construct four test statistics that are based upon the GLS detrended data, y_t^{GLS}. First define the term

$$\kappa = \sum_{t=2}^{T} (y_{t-1}^{GLS})^2 / T^2$$

The statistics may be written as:

$$\begin{aligned}
MZ_\alpha &= (T^{-1}(y_T^{GLS})^2 - f_0)/2\kappa \\
MSB &= (\kappa/f_0)^{1/2} \\
My_t &= MZ_\alpha \times MSB \\
MP_T &= \begin{cases} (\hat{c}^2\kappa - \hat{c}T^{-1}(y_T^{GLS})^2)/f_0 & \text{if } x_t = \{1\}, \\ (\hat{c}^2\kappa - (1-\hat{c})T^{-1}(y_T^{GLS})^2)/f_0 & \text{if } x_t = \{1, t\}, \end{cases}
\end{aligned}$$

where

$$\hat{c} = \begin{cases} -7 & \text{if } x_t = \{1\}, \\ -13.5 & \text{if } x_t = \{1, t\}, \end{cases}$$

and where $x_t = \{1\}$ and $x_t = \{1, t\}$ meaning that, under the alternative hypothesis, the time series is stationary about a constant or a constant and a linear time trend, respectively. In all cases, f_0 is substituted by an autoregressive estimate of the spectral density at frequency zero of v_t. In our application,

Table 1 Ng-Perron Unit Root Tests

Index	MZ_α	My_t	MSB	MP_T
H_1:*stationarity about a constant*				
Construction	-0.326	-0.261	0.803	35.644
Real Estate	-2.291	-1.069	0.467	10.688
H_1:*stationarity about a linear trend*				
Construction	-4.474	-1.351	0.302	19.286
Real Estate	-3.629	-1.229	0.339	23.325

The entries are the Ng-Perron (2001) statistics. The optimal lags were automatically selected by using the Modified Akaike Information Criterion. The frequency zero spectrum was estimated by the AR-GLS detrended data method. The superscript * indicates significance at the 10% significance level.

we follow Ng and Perron (2001) by computing the optimal lag length by means of the Modified Akaike Information Criterion.

The results displayed in Table 1 suggest that the null hypothesis of unit root cannot be rejected at standard significance levels. The overall conclusion would be that both indices are $I(1)$ processes.[6]

However, it is well known that unit root tests display low power against some type of stationary processes, for instance fractionally integrated processes.This is the main reason why there is the need to complement the results of unit root tests with results stemming from stationarity tests, as the KPSS test. The model that we use to test for stationarity of the time series, y_t, is similar to the one used by KPSS:

$$y_t = \alpha + \beta t + d \sum_{i=1}^{t} + \varepsilon_t,$$

$t = 1, \ldots, T$. The test statistic is constructed as follows: Regress y_t on deterministic components which consist either of a constant or of a constant and time trend by ordinary least squares. Denote the resulting residuals e_t, $t = 1, \ldots, T$, and compute

$$S_t = \sum_{s=1}^{t} e_s$$

The test statistic is then given by

$$\omega = T^{-2} \sum_{t=1}^{T} S_t^2 / \hat{\sigma}^2.$$

[6] Testing for one unit root in the first-differenced data allows us to strongly reject the null hypothesis of a second unit root. Results are available upon request.

Under the null hypothesis, $\hat{\sigma}^2$ is a consistent estimator of the long run variance of ε_t, σ^2_ε, which is defined by

$$\sigma^2_\varepsilon = \lim_{T\to\infty} T^{-1} E\left[\left(\sum_{t=1}^{T} \varepsilon_t\right)^2\right].$$

As suggested by Hobijn et al. (1998), we employ estimators which are frequently used in so-called heteroskedasticity and autocorrelation consistent (HAC) estimation. We employ the nonparametric approach that is based on estimators of the form

$$\hat{\sigma}^2 = \hat{\gamma}_0 + 2\sum_{j=1}^{T-1} k_m(j)\hat{\gamma}_j$$

where $\hat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} e_t e_{t-j}$ is used as the estimate of the j-th order autocovariance of ε_t, and $k_m(\cdot)$ is a kernel function depending on a bandwith parameter m. Several kernels have been proposed to weight the estimated autocovariances. We consider two of them: the Bartlett kernel and the Quadratic Spectral kernel (QS), because the Bartlett kernel is used by KPSS, while the Quadratic Spectral kernel has been shown by Andrews (1991) to be more efficient. Next, we have to choose the bandwidth, m. We apply two alternative data dependent procedures to estimate the optimal bandwidth parameter, as proposed by Andrews (1991) and Newey and West (1994), respectively.

Table 2 KPSS Tests (Andrews' Automatic Selection Lag)

Index	Bartlett kernel	QS kernel
H_0:*stationarity about a constant*		
Construction	0.359*	0.571**
Real Estate	0.281	0.543**
H_0:*stationarity about a linear trend*		
Construction	0.341***	3.715***
Real Estate	0.342***	2.824***

The entries are the Kwiatkovski et al. (1992) test statistics. The optimal truncation lag was automatically selected by using Andrews' (1991) procedure. The superscripts ** and *** indicate significance at the 5% and 1% significance level, respectively.

The results displayed in Tables 2 and 3 allow us to reject the null hypothesis of stationarity, either about a constant or about a linear time trend. This conclusion is maintained for different methods of computing the optimal truncation lag and for alternative types of kernels,[7] hence supporting the results obtained from the Ng-Perron unit root tests. Therefore, it could be argued that the UK Construction and Real Estate indices are unit root, or $I(1)$, processes.

[7] The evidence for Construction is weaker when using the Bartlett kernel.

Table 3 KPSS Tests (Newey-West's Automatic Selection Lag)

Index	Bartlett kernel	QS kernel
H_0:stationarity about a constant		
Construction	5.075***	10.651***
Real Estate	2.149***	4.503***
H_0:stationarity about a linear trend		
Construction	0.610***	1.259***
Real Estate	0.799***	1.668***

The entries are the Kwiatkovski et al. (1992) test statistics. The optimal truncation lag was automatically selected by using Newey-West's (1994) procedure. The superscripts ** and *** indicate significance at the 5% and 1% significance level, respectively.

4.2 Nonlinear trend stationarity

In the previous section we have provided evidence favourable to the unit root hypothesis for the Construction and Real Estate indices (in logs). The tests used in that section were constructed against the alternative hypothesis of *linear* stationarity, for instance, stationarity about a constant or a linear time trend. In this section we explore the possibility that the analysed time series could be stationary about a *nonlinear* time trend, including the case of stationarity about a changing constant.

Bierens' (1997) unit root tests against nonlinear trend stationarity are based on an Augmented Dickey-Fuller (ADF) type auxiliary regression model, using Chebishev polynomials to approximate nonlinear deterministic time trends. Specifically, Chebishev polynomials are defined as:

$$P_{0,n}(t) = 1, \quad P_{k,n}(t) = \left(\sqrt{2}\right) \cos\left[k\pi(t-0.5)/n\right],$$

for $t = 1, \ldots, n$ and $k = 1, \ldots, n-1$. Any smooth trend function $g(t)$ can be approximated by a linear combination of $m+1$ Chebishev polynomials:

$$g_{m,n}(t) = \sum_{k=0}^{m} \xi_{k,n} P_{k,n}(t) \approx g(t) \tag{1}$$

Since a linear time trend can be approximated for small values of m, Bierens (1997) proposes to orthogonalize the polynomials in order to distinguish between linear and nonlinear trends, using the following transformations:

$$\begin{aligned}
P^*_{0,n}(t) &= 1, \\
P^*_{1,n}(t) &= \frac{t-(n+1)/2}{\sqrt{(n^2-1)/12}}, \\
P^*_{2k,n}(t) &= \frac{P_{2k-1,n}(t) - \alpha_{k,n} - \sum_{j=1}^{k-1} \beta_{k,j,n} P_{2j-1,n} - \gamma_{k,n}(t/n)}{c_{k,n}}, \\
P^*_{2k+1,n}(t) &= P_{2k,n}(t),
\end{aligned}$$

Table 4 Bierens' (1997) Tests

Index	m	$t(m)$	$A(m)$	$F(m)$	$T_1(m)$	$T_2(m)$	$\tilde{T}(m)$
Construction	5	-3.599	-26.239	3.321	6.668	5.924	298.10
	10	-4.385	-41.554	3.351	16.527	15.783	1210.3
	15	-5.906	71.109	3.463	18.273	17.560	2213.9
	20	-6.922	-98.148	3.563	22.888	22.197	4385.5
Real Estate	5	-3.329	-23.931	4.137	13.506^{**}_{R}	12.330^{**}_{R}	588.79^{**}_{R}
	10	-4.110	-37.087	3.277	18.647^{*}_{R}	17.480^{*}_{R}	1512.5
	15	-6.441	-79.411	4.478	28.484^{**}_{R}	27.341^{**}_{R}	3920.1^{*}_{R}
	20	-7.395	-109.966	4.332	33.500^{**}_{R}	32.376^{**}_{R}	7805.2^{***}_{R}

m is the number of Chebishev polynomials. L and R stand for lef-tailed and right-tailed rejection, respectively. All the rejections with the $F(m)$ test are right-tailed. Superscripts indicate significance at two-sided levels of: *** 1%, ** 5% and * 10%.

for $k = 1, 2, \ldots, [n/2]$, where $[x]$ denotes the largest integer $\leq x$, $\alpha_{k,n}, \beta_{k,j,n}$ and $\gamma_{k,n}$ are the least squares coefficients of the regression of $P_{2k-1}(t)$ on 1, $P_{2j-1}(t)$, $j = 1, \ldots, k-1$, and t/n, respectively, and the $c_{k,n}$'s are norming constants such that $(1/n)\sum_{t=1}^{n} [P^*_{2k,n}(t)]^2 = 1$.

The ADF auxiliary regression model is:

$$\Delta y_t = \alpha y_{t-1} + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} + \theta^T P^{(m)}_{t,n} + \varepsilon_t, \tag{2}$$

where $P^{(m)}_{t,n} = (P^*_{0,n}(t), P^*_{1,n}(t), \ldots, P^*_{m,n}(t))^T$. Bierens (1997) develops six testing procedures:

1. $\hat{t}(m)$, the t-ratio statistic of $\hat{\alpha}$.
2. $\hat{A}(m) = n\hat{\alpha}/|1 - \sum_{i=1}^{p} \phi_i|$.[8]
3. $\hat{F}(m)$, joint F test on $\hat{\alpha}$ and the coefficients of non-constant Chebishev polynomials, $P_{j,t}$, $j = 1, \ldots, m$.
4. $\hat{T}_1(m)$, joint chi-squared test on $\hat{\alpha}$ and the coefficients of non-constant Chebishev polynomials, $P_{j,t}$, $j = 1, \ldots, m$.
5. $\hat{T}_2(m)$, joint chi-squared test on $\hat{\alpha}$ and the coefficients of non-constant Chebishev polynomials, $P_{j,t}$, $j = 2, \ldots, m$.
6. $\tilde{T}(m)$, nonparametric version of $\hat{T}_2(m)$.

See Bierens (1997) for further details about computation and critical values of each test. Left and right-sided rejection of the null hypothesis provides information about the linear or nonlinear nature of the stationary trend. Table **??** summarises the possibilities (this table is taken from Cushman, 2002, Table 1, p. 3)

However, as pointed out by Bierens (1997) and Cushman (2002), right-sided rejection of $t(m)$, $A(m)$, $T_2(m)$ and $\tilde{T}(m)$ could be the result of unit root processes with nonlinear drift.

For each data set, we assume that the null of the unit root with constant drift hypothesis is true; then, an autoregression for the first difference of the

[8] As Cushman (2002) points out, this formula differs from that in Bierens (1997) by taking the absolute value in the denominator.

Table 5 Bierens' (1997) Tests

Index	m	$t(m)$	$A(m)$	$F(m)$	$T_1(m)$	$T_2(m)$	$\tilde{T}(m)$
Construction	5	-3.599	-26.239	3.321	6.668	5.924	298.10
	10	-4.385	-41.554	3.351	16.527	15.783	1210.3
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m is the number of Chebishev polynomials. L and R stand for lef-tailed and right-tailed rejection, respectively. All the rejections with the $F(m)$ test are right-tailed. Superscripts indicate significance at two-sided levels of: *** 1%, ** 5% and * 10%.

time series is fitted with a constant and a lag choice determined using the Akaike criterion with a maximum lag length equal to 10. For each series and using the estimated parameters and the estimated residual variance, we compute a bootstrap first differenced series, which is accumulated. The unit root tests is applied using the wild bootstrap approach, using the Akaike criterion for lag length choice. The procedure is repeated 1000 times for each series, and the empirical one-sided p-values computed by comparison of the original tests values with the bootstrap distributions.

Results for $t(m)$, $A(m)$, $F(m)$, $T_1(m)$, $T_2(m)$ and $\tilde{T}(m)$ tests are shown in Table 5.[9] The null hypothesis of unit root cannot be rejected for the Construction index, however, there is strong evidence against the null hypothesis in the case of the Real Estate index for $m > 5$. Moreover, the right-sided rejections of the $T_1(m)$, $T_2(m)$ and $\tilde{T}(m)$ tests would allow us to conclude that the Real Estate index is stationary about a nonlinear time trend. Figure 1 displays both the original time series, in logs, and the estimated nonlinear time trend by using $m = 20$ Chebishev polynomials; note the nonlinearity of the long term evolution of the Real Estate index, and the stationary deviations of the daily values about such a nonlinear trend.

5 Random walk tests

The variance ratio test proposed by Lo and MacKinlay (1988, 1989) is based on the fact that, for a random walk series, the variance of its k-th difference is k times the variance of its first difference. For example, if a series follow a random walk, the variance of its four-day difference will be four times as large as the variance of its daily difference. The hypothesis to be tested is H_0: the time series follows a random walk, vs. H_1: the time series does not follow a random walk.

In a recent paper, Wright (2000) proposes the use of signs and ranks of differences in place of the differences in the Lo and MacKinlay tests. Wright

[9] These test statistics, with the exception of $T_1(m)$ and $T_2(m)$, can be computed using the program EasyReg, written by Herman J. Bierens and freely available at the URL http://econ.la.psu.edu/ hbierens/EASYREG.HTM

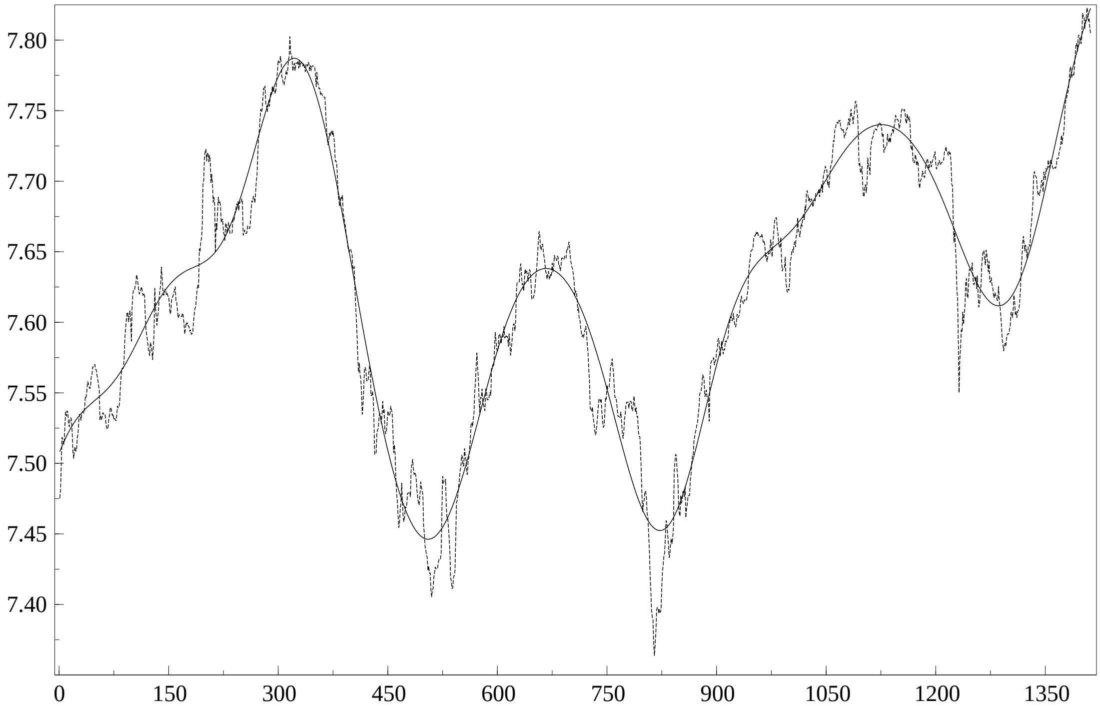

Fig. 1 Nonlinear time trend (dashed line) of Real Estate index (in logs, continuous line) approximated by $m = 20$ Chebishev polynomials.

demonstrates that his nonparametric variance ratio tests based on ranks (R_1 and R_2) and signs (S_1 and S_2), can be more powerful than the tests suggested by Lo and MacKinlay. They have high power against a wide range of models displaying serial correlation, including fractionally integrated alternatives. The tests based on ranks are exact under the independence and identical distribution assumption, wheras the tests based on signs are exact even under conditional heteroskedasticity. Moreover, Wright (2000) shows that ranks-based tests display low size distortion, under conditional heteroskedasticity.

Given T observations of asset returns $\{y_1, \ldots, y_T\}$, Wright's proposed R_1 and R_2 are defined as:

$$R_1 = \left(\frac{\frac{1}{Tk} \sum_{t=k}^{T} \left(r_{1t} + \ldots + r_{1t-k+1} \right)^2}{\frac{1}{T} \sum_{t=1}^{T} r_{1t}^2} - 1 \right) \times \phi(k)^{-1/2}, \tag{3}$$

$$R_2 = \left(\frac{\frac{1}{Tk} \sum_{t=k}^{T} \left(r_{2t} + \ldots + r_{2t-k+1} \right)^2}{\frac{1}{T} \sum_{t=1}^{T} r_{2t}^2} - 1 \right) \times \phi(k)^{-1/2}, \tag{4}$$

where

$$r_{1t} = \left(r\left(y_t\right) - \frac{T+1}{2} \right) \Bigg/ \sqrt{\frac{(T-1)(T+1)}{12}},$$
$$r_{2t} = \Phi^{-1}\left(r(y_t)/(T+1)\right).$$

$\phi(k)$ is defined in (5), $r(y_t)$ is the rank of y_t among $y_1, \ldots, y_T$, and Φ^{-1} is the inverse of the standard normal cumulative distribution function. The tests based on the signs of returns are given by:

$$S_1 = \left(\frac{\frac{1}{Tk}\sum_{t=k}^{T}\left(s_t + \ldots + s_{t-k+1}\right)^2}{\frac{1}{T}\sum_{t=1}^{T} s_t^2} - 1 \right) \times \phi(k)^{-1/2}, \tag{5}$$
$$S_2 = \left(\frac{\frac{1}{Tk}\sum_{t=k}^{T}\left(s_t(\overline{\mu}) + \ldots + s_{t-k+1}(\overline{\mu})\right)^2}{\frac{1}{T}\sum_{t=1}^{T} s_t(\overline{\mu})^2} - 1 \right) \times \phi(k)^{-1/2}, \tag{6}$$

where $\phi(k)$ is defined in (5), $s_t = 2u(y_t, 0)$, $s_t(\overline{\mu}) = 2u(y_t, \overline{\mu})$, and

$$u(x_t, q) = \begin{cases} 0.5 & \text{if } x_t > q, \\ -0.5 & \text{otherwise.} \end{cases}$$

Thus, S_1 assumes a zero drift value. If the value of the drift parameter is unknown, the procedure described in Luger (2003), based on Campbell and Dufour (1997), is applied to compute S_2. This method consists of a two-step strategy.

First, an exact confidence interval for the drift parameter μ, valid under the null hypothesis, is established. Denote $y_{(1)}, \ldots, y_{(T)}$ the order statistics of the sample $y_1, \ldots, y_T$. An exact confidence interval $CI_\mu(\alpha_1)$ for μ with level $1 - \alpha_1$ is given by $\left[y_{(h+1)}, y_{(T-h)}\right]$, where h is the largest integer such that $\Pr\left[B \leq h\right] \leq \alpha_1/2$, for B a binomial random variable with number of trials T and probability of success $1/2$. The second step consists of computing the S_2 statistic, for each candidate value b for the drift parameter in the confidence interval. The value of the S_2 statistic (retaining the sign) at aggregation interval k is then defined as

$$S_2(k) = \inf\left\{|S_2(k, b)| : b \in CI_\mu(\alpha_1)\right\}$$

where, given $b \in CI_\mu(\alpha_1)$, $S_2(k, b)$ is computed by defining $s_t(b) = 2u(y_t, b)$. The chosen S_2 value is compared to the appropriate critical values for an α_2 level test, such that the overall level of the strategy is bounded by $\alpha = \alpha_1 + \alpha_2$. In this paper, we have set $\alpha_1 = 0.01$ and $\alpha_2 = 0.04$.

However, as pointed out by Wright (2000) and shown in Belaire-Franch and Opong (2005a), using several k values would lead to an over rejection of the null hypothesis, as in Lo and MacKinlay's tests context. Thus, Belaire-Franch and Contreras (2004) suggest several ranks and signs-based multiple variance ratio tests in order to get control of the final size. One set of statistics consists of applying p-value adjustments for multiplicity based on simple bootstrap, in line with Psaradakis (2000).

The goal of the procedure is to obtain an approximation to the null sampling distribution of $\min_{1\leq i\leq m} p_{ji}$, where p_{ji} is the p-value corresponding to the variance ratio test j computed for an individual k value, and m is number of k values, as follows. First, one simulates N bootstrap samples, each of size T, by resampling with replacement from the original first differences. Then, for the nth bootstrap sample, we compute the value of the m variance ratio test statistics and the associated p-values $p^*_{j1,n}, \ldots, p^*_{jm,n}$, repeating the same process for $n = 1, \ldots, N$, obtaining the sample $\{\min_{1\leq i\leq m} p^*_{ji,n} : n = 1, \ldots, N\}$. The empirical distribution of $\{\min_{1\leq i\leq m} p^*_{ji,n} : n = 1, \ldots, N\}$ is an estimate of the bootstrap approximation to the sampling distribution of $\min_{1\leq i\leq m} p_{ji}$ under the null hypothesis of independent and identically distributed ($i.i.d.$) increments. Bootstrap-adjusted p-values are computed as

$$\tilde{p}^{(N)}_{ji} = \frac{1}{N}\sum_{n=1}^{N} I_{(-\infty,0]}\left(\min_{1\leq l\leq m} p^*_{jl,n} - p_{ji}\right), \ \ i = 1, \ldots, m$$

where $I_A(\omega)$ is an indicator function, equal to 1 if $w \in A$ and 0 otherwise. We reject the null hypothesis with test j if $\min_{1\leq i\leq m} \tilde{p}^{(N)}_{ji} \leq \alpha$. Note, however, that the bootstrap adjustment would be valid for the $R^{(\cdot)}_1$ and $R^{(\cdot)}_2$ tests only under the more restrictive assumption of $i.i.d.$ differences, whereas it remains reliable for the $S^{(\cdot)}_1$ and $S^{(\cdot)}_2$ tests under the uncorrelated and heteroskedastic increments case. Belaire-Franch and Opong (2005a) show that the simple bootstrap adjustment for multiplicity leads to signs-based tests with size close to the significance level.

Table 6 Signs-based Variance Ratio Tests for U.K. Construction and Real Estate Indices

	Construction	Real Estate
S_1		
$k = 2$	5.129***	3.243***
$k = 4$	4.667***	3.346***
$k = 8$	3.876***	2.868***
$k = 16$	3.065***	2.926***
S_2		
$k = 2$	4.752***	3.243***
$k = 4$	4.404***	3.346***
$k = 8$	3.372***	2.868***
$k = 16$	2.203***	2.679***
Multiple bootstrapped S_1 test p-value		
$k = 2, 4, 8, 16$	0.000***	0.000***
Multiple bootstrapped S_2 test p-value		
$k = 2, 4, 8, 16$	0.000***	0.000***

Superscript *** indicates significance at the 1% level.

The results in Table 6 suggest that the null hypothesis of uncorrelated increments is clearly rejected for each value of the lag parameter k. This conclusion is strongly supported by the multiple signs-based tests. This would imply that the inidices returns are linearly predictable. However, the presence of significant linear autocorrelation cannot be interpreted as the existence of a linear data generating process. The next section explores alternative explanations for the evolution of the indices returns.

6 Nonlinearity and chaos tests

In this section, we assess the hypothesis of nonlinear dynamics and chaos for the indices returns. Both hypothesis are closely related, since nonlinearity is a necessary condition for deterministic chaos. Hence, if the linearity hypothesis is rejected, addtional testing procedures are needed to conclude that the corresponding dynamical system is chaotic.

In order to test for the linearity hypothesis, we apply two alternative procedures. On the one hand, we compute the Kaplan's (1994) test and, on the other hand, we compute Terasvirta et al. (1993) test with wild bootstrap as suggested by Becker and Hurn (2004). We compute two tests because, as argued by Barnett el al. (1997), different procedures might be sensitive to different deviations from the null hypothesis of linearity.

In order to test for chaos, we take into account the fact that the largest Lyapunov exponent of a chaotic deterministic system is positive. Then, we apply Shintani and Linton (2004) test for the positivity of the Lyapunov exponent.

6.1 Nonlinearity tests

In this section, we apply two alternative procedures to test for linearity in the Real Estate and Construction indices returns. First, we compute Kaplan's (1994) test. The Kaplan test compares a test statistic computed directly from the data with the test statistic produced from surrogate data.

The method of surrogate data (Theiler et al., 1992) consists of taking a Fourier transform of the raw data, keeping the original magnitudes and randomising the phases: the resulting inverse Fourier transform contains the same linear correlations as the original data. In the present work, the iteratively amplitude adjusted Fourier transform (IAAFT) algorithm, suggested by Schreiber and Schmitz (1996) is used. This method generates surrogate data sets with the same linear correlations and the same probability distribution as the original data.

Regarding the Kaplan test, it is based on the fact that deterministic solution paths have the property that indicates that points that are nearby are also nearby under their image in phase space. Kaplan's statistic has a strictly positive lower bound for a stochastic process but not for a deterministic solution path.

Table 7 Kaplan (1994) Test

Index	m	K	Min K on surrogates	Conclusion
Construction	2	0.00144	0.00147	Reject linearity
	3	0.00122	0.00141	Reject linearity
	4	0.00107	0.00149	Reject linearity
	5	0.00104	0.00147	Reject linearity
Real Estate	2	0.00285	0.00220	Accept linearity
	3	0.00154	0.00210	Reject linearity
	4	0.00151	0.00217	Reject linearity
	5	0.00118	0.00212	Reject linearity

K is the Kaplan test. Twenty surrogates were used. Hence, the minimun is over the 20 surrogates. Embedding dimension, m, as defined by Kaplan, is $m-1$, whereas embedding dimension is defined as in Nychka et al. (1992). The lag parameter τ has been set equal to 1.

Given the time series y_t, $t = 0, \dots, T$, and an embedding dimension m, we define the so-called embedding vectors (or m-histories)

$$y_t^m = (\, y_t \; y_{t+\tau} \; y_{t+2\tau} \; \cdots \; y_{t+(m-1)\tau} \,),$$

$t = 1, ..., T-(m-1)\tau$, where τ is the time delay. There is a recursive function $y_{t+\tau} = f(y_t^m)$, where $y_{t+\tau}$ is called the 'image' of the point y_t^m in phase space. For deterministic systems with a continuous f, nearby points in m-dimensional phase space will have nearby images, whereas for a stochastic system nearby points in phase space may have very different images.

Formally, this amounts to testing whether for pairs of data points which are within some small distance $d_{ij} = |y_i^m - y_j^m| < r$, the average of the differences of their iterations $\varepsilon_{ij} = |y_{i+1}^m - y_{j+1}^m|$ is found to be smaller than some threshold value. We performed 20 replications with surrogate data and computed the test statistic K as the average ε_{ij} from the 500 smallest distances d_{ij}. The resulting test statistic K is compared to the minimum K from 20 time series of surrogate data. With the latter greater (smaller) than the actual one, we accept (reject) linearity of the data.

Table 7 presents the results. In all cases, we have computed the test statistics for m ranging from 2 through 5, and the lag parameter τ has been set equal to 1. For all the embedding dimensions, but one, the null hypothesis of linearity is rejected for both indices returns.

Hence, Kaplan's test results point towards a nonlinear data generating process, althoug we cannot infer whether the process is deterministic or stochastic. Moreover, as shown in Barnett et al. (1997), Kaplan's test is powerful against nonlinear in variance processes as well. Then, we should perform additional tests in order to distinguish between nonlinearity in mean and nonlinearity in variance, and between stochastic and deterministic nonlinearity.

The Neural Network test for neglected nonlinearity by White (1989) is based on the fact that, under the null hypothesis of linearity in the mean for a variable y_t,

$$E[u_t \Psi_t] = 0$$

where u_t is the random term of the linear model and Ψ_t is any function that depends only on variables defined in $\mathscr{F}_{t-1}$. [10]In White (1989) and Lee et al. (1993),

$$\Psi_t = (\psi(y_{t-1}\Gamma_1), \psi(y_{t-1}\Gamma_2), \ldots, \psi(y_{t-1}\Gamma_q)),$$

where $\Gamma = (\Gamma_1, \Gamma_2, \ldots, \Gamma_q)$ is a set of randomly chosen parameters, independent of y_t, and ψ is the logistic cumulative distribution function.

Terasvirta et al. (1991) propose using an LM-type version of the neural network test based on a Volterra expansion of the nonlinear function. They call the resulting procedure V23, and it involves the following steps:

1. Regress y_t on 1 and p lags, and compute the residuals, $\hat{u}_t$ and the sum of squared residuals, $SSR_0 = \sum \hat{u}_t^2$.
2. Regress $\hat{u}_t$ on 1 and $y_{t-1}, \ldots, y_{t-p}$ and m auxiliary regressors in the form:

$$\hat{u}_t = \pi' y_t + \sum_{i=1}^{p}\sum_{j=i}^{p} \delta_{ij} y_{t-i} y_{t-j} + \sum_{i=1}^{p}\sum_{j=i}^{p}\sum_{k=j}^{p} \delta_{ijk} y_{t-i} y_{t-j} y_{t-k} + \omega_t$$

   where $\pi' = (\pi_0\ \pi_1 \ldots \pi_p)$ and $y_t = (1\ y_{t-1} \ldots y_{t-p})'$. Compute the residuals $\hat{\omega}_t$ and the residual sum of squares $SSR = \sum \hat{\omega}_t^2$.
3. Compute

$$F = \frac{(SSR_0 - SSR)/m}{SSR/(T-p-1-m)}$$

   which is approximately $F_{m,T-p-1-m}$ distributed under the null hypothesis of linearity in the mean.

Nevertheless, conditional heteroskedasticity can seriously affect the size of the neural network tests. Thus, if the model for the variance is misspecified there is “remaining” heteroskedaticity. Neural network tests results could be reflecting this issue.

In this section, however, we apply Terasvirta et al. (1991) test on the original returns series to test the null hypothesis of linearity in mean. To account for conditional heteroskedasticity, we apply a fixed-design wild bootstrap (WB) approach, following Becker and Hurn’s (2004) suggestion and Davidson and Flachaire’s (2000) WB procedure, as follows:

1. First, estimate the model under the null of linearity, i.e.:

$$y_t = \pi_0 + \pi_1 y_{t-1} + \ldots + \pi_p y_{t-p} + u_t$$

   where p is chosen using the Schwarz information criterion.
2. Compute the residuals $\hat{u}_t$ and the transformed residuals $\hat{u}_t^* = \hat{u}_t/(1-h_t)$, where $h_t = \mathbf{y_t}(\mathbf{Y'Y})^{-1}\mathbf{y_t'}$, and where $\mathbf{y_t}= (1\ y_{t-1}\ \ldots\ y_{t-p})$.
3. Next, build the time series $\hat{u}_t^* \epsilon_t$, where ϵ_t is an IID random variable defined as:

$$\epsilon_t = \begin{cases} 1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2 \end{cases}$$

[10] $\mathscr{F}_{t-1}$ is the information set at time $t-1$.

4. Using the estimated parameters in the first step, simulate an AR(p) model for the series, using as pseudo-random disturbances the transformed residuals $\hat{u}_t^* \epsilon_t$.
5. For each replication computed in the previous step, compute the V23 test. Perform this procedure a large number of times.
6. Compute the empirical p-value of the original time series using the empirical distribution under unknown heteroskedasticity.

Becker and Hurn show that their procedure is robust against GARCH-type conditional heteroskedasticity.[11] Hence, rejection of the null hypothesis using the V23 test with wild bootstrap could not be attributed to conditional heteroskedasticity of the GARCH class.

Table 8 Terasvirta et al. (1993) Test

	Construction	Real Estate
statistic		
V23 asymptotic p-value	0.000	0.000
WB p-value	0.803	0.369

Wild bootstrap p-values based on 1000 replications.

Table 8 shows that the null hypothesis of linearity in mean is strongly reject when using the asymptotic distribution of the original V23 test. However, if we compute the WB distribution, the evidence vanishes for both indices.

6.2 Chaos tests

From the results in the previous section, we could conclude that both indices returns are nonlinear, although nonlinearity in the case of the Real Estate returns is probably due to nonlinearity in variance. Nevertheless, in the case of the Construction returns we have found evidence against linearity in the mean.

In this section we test the hypothesis of deterministic chaos. It is well known that deterministic chaos is the result of certain kind of nonlinear deterministic dynamical systems, which are called chaotic systems. Time series generated by this type of systems look like stochastic despite the fact that they are purely deterministic.

The Lyapunov exponent, which measures the average rate of divergence or convergence of two nearby trajectories, is a useful measure of the stability of a dynamic system. In fact, the maximum Lyapunov exponent of a chaotic system is positive. Traditionally, reaserchers have computed point estimates for the maximum Lyapunov exponent by means of alternative procedures (e.g., Wolf et al., 1985; Nychka et al., 1992; Gencay and Dechert, 1992; Rosenstein

[11] As noted by these authors, however, the robustness of the tests to GARCH cannot automatically be taken to extend to other types of heteroskedasticity.

et al., 1993; among others.) More recently, statistical theory has been developed by Whang and Linton (1999) and Shintani and Linton (2004), to deal with the statistical testing of the hypothesis of deterministic chaos, in the context of nonparametric kernel-type and neural network estimation of Lyapunov exponents, respectively.

We will test for deterministic chaos by computing the Jacobian-based estimator using neural network nonparametric regression, as proposed by Nychka et al. (1992) and using the statistical framework of Shintani and Linton (2004).

Let $\{y_t\}_{t=1}^{T}$, be a random scalar sequence generated by the nonlinear autoregressive model:

$$y_t = \theta_0(y_{t-1}, \dots, y_{t-d}) + u_t$$

where $\theta_0 : \mathbb{R}^d \to \mathbb{R}$ is a nonlinear dynamic map and $\{u_t\}$ is a sequence of random variables. The model can be expressed in terms of a map with an error vector $U_t = (u_t, 0, \dots, 0)$ and the map function $F : \mathbb{R}^d \to \mathbb{R}^d$ such that

$$Z_t = F(Z_{t-1}) + U_t$$

where $Z_t = (y_t, \dots, y_{t-d+1})' \in \mathbb{R}^d$. Let J_t be the Jacobian of the map F evaluated at Z_t:

$$J_t = \begin{bmatrix} \Delta\theta_{01t} & \Delta\theta_{02t} & \dots & \Delta\theta_{0d-1t} & \Delta\theta_{0dt} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

for $t = 0, 1, \dots, T-1$, where $\Delta\theta_{0jt} = D^{e_j}\theta_0(Z_t)$ for $j = 1, \dots, d$ and $e_j = (0, \dots, 1, \dots, 0)' \in \mathbb{R}^d$ denotes the jth elementary vector. Given a nonparametric neural network estimator of θ_0, $\hat{\theta}$, we can obtain $\hat{J}_t$ by substituting $\hat{\theta}$ in the J_t matrix. Then, the neural network estimator of the largest Lyapunov exponent is given by

$$\hat{\lambda}_M = \frac{1}{2M} \ln \upsilon \left(\hat{T}_M' \hat{T}_M \right), \quad \hat{T}_M = \prod_{t=1}^{M} \hat{J}_{M-t} = \hat{J}_{M-1} \cdot \hat{J}_{M-2} \cdots \hat{J}_0$$

where $\upsilon(A)$ is the largest eigenvalue of a matrix A. Now we distinguish between the sample size T used for estimating the Jacobian $\hat{J}_t$ and the block length M, which is the number of evaluation points used for estimating the Lyapunov exponent.

Our interest in this section is to test the null hypothesis $H_0 : \lambda \geq 0$ (deterministic chaos) against the alternative $H_1 : \lambda < 0$, therefore the test is one-sided. Given a consistent estimate of the unknown population standard deviation of the estimator $\hat{\lambda}$, $\hat{\Phi}$, the test statistic is given by

$$\hat{t} = \frac{\hat{\lambda}_M}{\sqrt{\hat{\Phi}/M}}$$

Under the null hypothesis, $\hat{t}$ is asymptotically distributed as a $N(0,1)$ random variate. In this paper, we compute the heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator, as suggested by Shintani and Linton (2004).[12]

Results are displayed in Table 9. In order to take into account the sensitivity of the results to the choice of the block lenght M, three alternative selection procedures have been applied: Full, Block and ES (equally spaced subsamples, see Shintani and Linton, 2004, for the details). In all cases, the lag length (d) and the number of hidden units (r) of the neural network have been jointly selected based on the Bayesian information criterion. The quadratic spectral kernel with optimal bandwidth (Andrews, 1991) has been used for the heteroskedasticity and autocorrelation consistent covariance estimation. In all cases, the null hypothesis of deterministic chaos is strongly rejected, whatever the block lenght choice, for both indices returns. In the case of the Real Estate index returns, this result is consistent with the lack of evidence of nonlinearity in the mean given in the previous section, since nonlinearity in the mean is a necessary condition, though not sufficient, for deterministic chaos. Moreover, the Construction index returns would be the result of a nonlinear-in-mean stochastic or deterministic process, but not chaotic.

Table 9 Shintani and Linton (2004) Test

	(d,r)	Full	Block	ES
Construction returns	(1,1)	- 2.106 (-157.638^{***})	-2.059 (-38.753^{***})	-2.107 (-37.227^{***})
Real Estate returns	(1,1)	-2.988 (-95.223^{***})	-2.758 (-34.314^{***})	-3.010 (-25.031^{***})

Superscript *** indicates rejection of the chaos hypothesis at the 1% level. For the full sample estimation (Full), the largest Lyapunov exponent estimates are presented with $\hat{t}$ statistics in parentheses for $H_0 : \lambda \geq 0$. For the estimation based on blocks (Block) and equally spaced subsamples (ES), median values are presented. The block length (M) for subsample is 87. The lag length (d) and the number of hidden units (r) are jointly selected based on BIC. Quadratic Spectral kernel with optimal bandwidth (Andrews, 1991) is used for the heteroskedasticity and autocorrelation consistent covariance estimation.

7 Nonlinear forecastability test

In this section we look for nonlinear deterministc structures in Real Estate and Construction indices returns by means of a nonlinear forecastability test. More specifically, we compute the Finkenstadt and Kuhbier (1995) test.[13]

Given the time series y_t, $t = 1, \ldots, T$, the following forecasting algorithm is applied:

[12] We are grateful to Professor Shitani for providing the computer code to compute the statistical tests concerning Lyapunov exponents, as described in Shintani and Linton (2004).

[13] Hereafter, FK test.

1. The data set $\{y_t\}_{t=1}^T$ is divided into two parts, the fitting set $F = \{y_1, \ldots, y_{T_f}\}$ and the testing set $T = \{y_{T_f+1}, \ldots, y_{T_f+T_t}\}$. Choose and embedding dimension m and construct the embedded vectors for each set. Let F^m and T^m denote the set of m-dimensional vectors of the corresponding part of the data set.
2. For each m-history in the testing set y_i^m, compute the distances to the m-histories in the fitting set. Following Sugihara et al. (1991) and Finkenstadt and Kuhbier (1995), find the $m+1$ nearest neighbors $y_{i_k}^m$, $k = 1, \ldots, m+1$.
3. Use the distances to compute exponential weights, where neighbors with a larger distance are assigned lower weights

$$w_k^i = \frac{e^{|y_i^m - y_{i_k}^m|}}{\sum_{k=1}^{m+1} e^{|y_i^m - y_{i_k}^m|}},\ k = 1, \ldots, m+1.$$

4. Follow the evolution of each neighbor t periods later and multiply the value by the weight corresponding to that neighbor. The prediction for y_{i+t} is then obtained by the linear combination:

$$\hat{y}_{i+t} = \sum_{k=1}^{m+1} w_k^i y_{i_k},\ i = T_f + m, \ldots, T_f + T_t - t.$$

Let $Y_t = \{\hat{y}_{T_f+1+t}, \ldots, \hat{y}_{T_f+T_t}\}$ be the set of predicted values and let $X_t = \{y_{T_f+1+t}, \ldots, y_{T_f+T_t}\}$ denote the set of observed values. In order to evaluate the nonlinear forecastability of the algorithm, Finkenstadt and Kuhbier (1995) suggest computing the Spearman rank correlation coefficient r_s between the values in the sets Y_t and X_t. The null hypothesis of linearity is stated as $H_0 : r_s(X_t, Y_t) = 0$, whereas the alternative is $H_1 : r_s(X_t, Y_t) > 0$. Under the null hypothesis, $t = \sqrt{T-1} r_s \sim N(0,1)$ and we use the one-sided critical values of the standard normal distribution. If the time series is the outcome of a deterministic, maybe chaotic, system, we would expect a positive and statistically significant correlation coefficient for low values of the prediction time. Forecastability would be confined to the very short term, hence the correlation coefficient would become non-significant or even negative for larger values of the prediction time.

The results of this test, however, are sensitive to the existence of autocorrelation and to the choice of the embedding dimension m. Then, we compute the statistic for the prewhitened time series, by means of a linear AR(p) model, and for several values of the embedding dimension. In our application, the order p of the AR model has been chosen by the Akaike Information Criterion, and the embedding parameter ranges from 2 through 20. The statistics with the 5% and 10% critical values are represented in Figures 2 and 3. The larger prediction time has been set equal to 5 days. The results shown in these figures correspond to the m values which displayed stronger evidence against the null hypothesis.

Hence, the FK test for the Construction index returns, Figure 2 would allow us to reject the null hypothesis at different significance levels and different

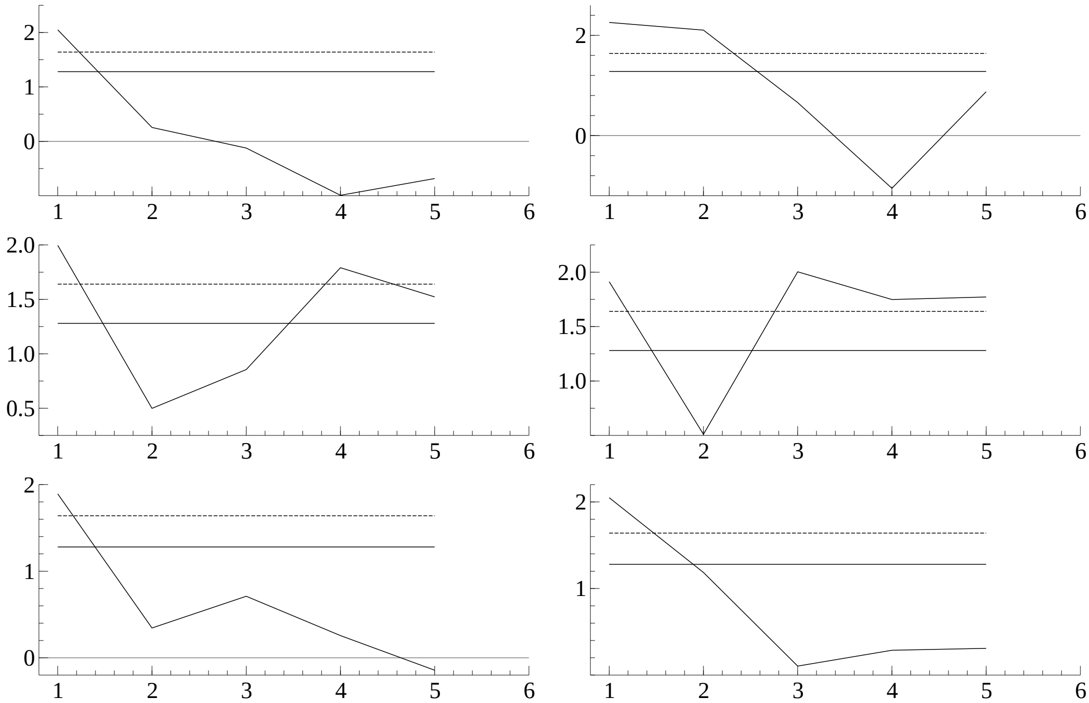

Fig. 2 FK test plotted against prediction time for filtered U.K. Construction returns for embedding dimension $m = 2, 4, 6, 7, 8, 17$. The horizontal lines correspond to the asymptotic 5% and 10% critical values.

embedding dimensions, although in general, forecastability does not extend further than three periods ahead. Nevertheless, the FK test for the Real Estate index returns, Figure 3, would allow us to (marginally) reject the null just for one value of the m parameter.

As a conclusion, given the results in the previous sections, we could claim that the Construction index returns time series is nonlinear in mean (and probably in variance), maybe as the result of a nonlinear deterministic, but not chaotic, process. The Real Estate index returns time series, however, is linear in mean and nonlinear in variance.

8 Conclusions

In this paper we have analysed the behaviour of the UK Real Estate and Construction indices. When analysing the levels of the time series, the unit root hypothesis cannot be rejected using recent unit root and stationarity tests. However, if nonlinear time trends are allowed, the hypothesis of unit root is clearly rejected in the case of the Real Estate index. Hence, we could claim

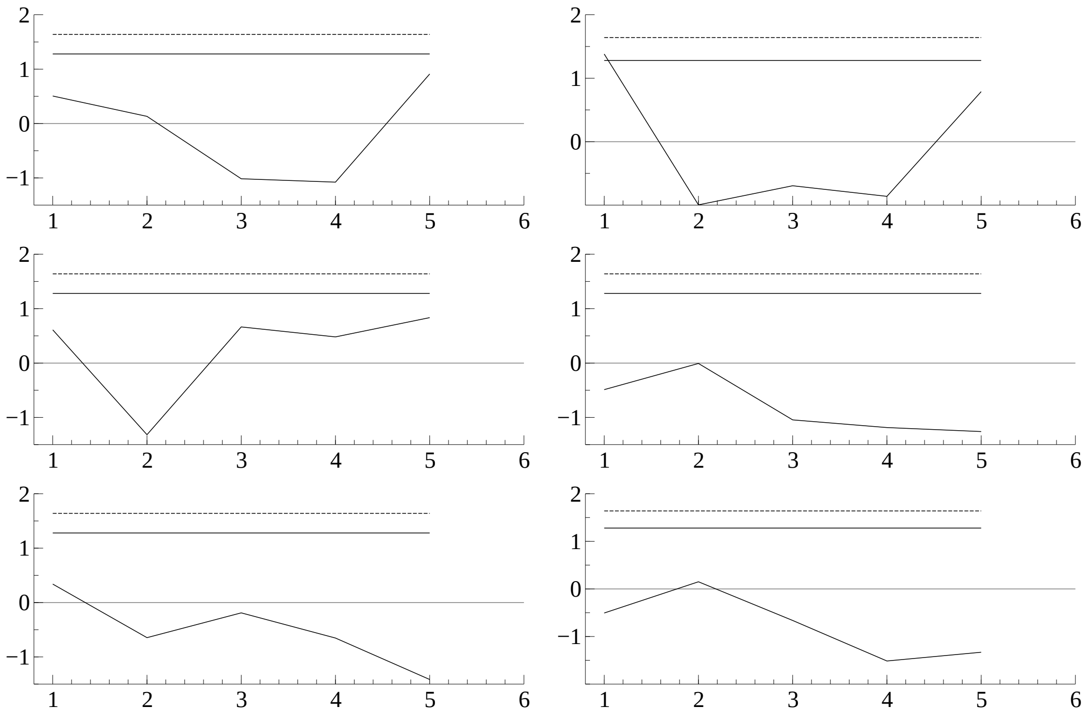

Fig. 3 FK test plotted against prediction time for filtered U.K. Real Estate returns for embedding dimension $m = 2, 3, 4, 5, 6, 7$. The horizontal lines correspond to the asymptotic 5% and 10% critical values.

that the Construction index is a unit root process whereas the Real Estate index is a stationary process about a nonlinear time trend.

On the other hand, we have analysed the behavior of the indices returns. We have found strong evidence against the hypothesis of linearly uncorrelated returns, although this cannot be taken as evidence of linear structure in the indices returns. In fact, the null hypothesis of linearity in mean of the Construction index returns is rejected. Shintani and Linton's (2004) test rejects the hypothesis of deterministic chaos for both indices returns, but we have found evidence of nonlinear forecastability for the Construction index returns. Hence, we could conclude that the first difference of the Construction index is a nonlinear in mean process, but not chaotic.

Our conclusions are very important from the point of view of both the modelling of the indices and the forecasting of their evolution. Hence, we have shown that the levels of the Real Estate index could be modelled as a stationary process about a nonlinear time trend, whereas the returns of the Construction index can be modelled as a nonlinear in mean process (for instance, by using Neural Networks or radial basis functions.) Therefore, given the knowledge of the nonlinear time trend, we would be able to perform predictions about the levels of the Real Estate index spanning a relatively long time period. Moreover, we have shown that we could outperform the forecasting ability of a linear AR model when performing short term predictions about the returns of the Construction index, by means of nonlinear forecasting techniques.

It is very important to note that our conclusions above have not taken into account the economic exploitability of our findings. The RWH may therefore still be valid given transaction costs. Nonetheless, our findings are important and cast doubt on earlier studies that were based on linear modelling which suggested that the behaviour of asset prices in the securities market was weak form efficient. The challenge now is to identify an exploitable trading strategy on the basis of predictions that could be formed based on nonlinear modelling which our findings suggest and which takes into account all transaction costs. It may also be that any perceived excess returns based on identified trading strategy may not be large enough to off-set transaction costs. Therefore, the search for an exploitable trading strategy based on our results may be worth having but it may not be worth searching for. This is an empirical challenge that needs to be addressed for the future.

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